BAULKHAM HILLS HIGH SCHOOL

2024

YEAR 12 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks: 100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 13)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1 Consider the following statement

"If the volleyball team doesn't play well, then they are not training enough"

Which of the following is the contrapositive of this statement?

(A) If they are not training enough, then the volleyball team doesn't play well.

(B) If they are training enough, then the volleyball team plays well.

(C) If the volleyball teamplays well, then they are training enough.

(D) If they train enough, then the volleyball team will most likely win.

2 $\int f(x)\sin x\,dx = -f(x)\cos x + 3\int x^2 \cos x\,dx$

Which of the following could be $f(x)$?

(A) x^3

(B) $-x^3$

(C) $3x^2$

(D) $-3x^2$

3 Which one of the following relations does **NOT** have a locus that is a straight line passing through the origin?

(A) $z = i\,\overline{z}$

(B) $z + \overline{z} = 0$

(C) $\text{Re}(z) - 2\text{Im}(z) = 0$

(D) $\text{Re}(z) + \text{Im}(z) = 1$

4 Which of the following is equivalent to

$$\frac{e^{-\frac{i\pi}{2}}}{e^{\frac{i\pi}{6}}} \text{ ?}$$

(A) $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

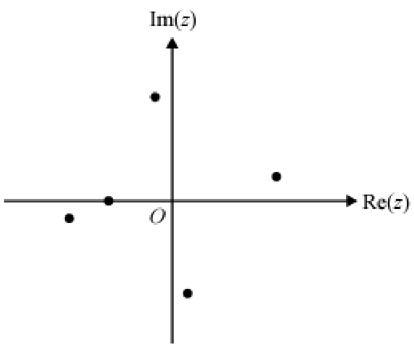
(B) $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

(C) $\frac{1}{2}-\frac{\sqrt{3}}{2}i$

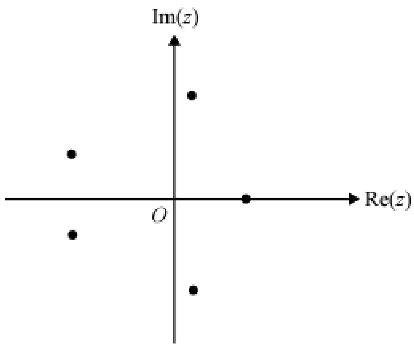
(D) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$

5 Which of the following diagrams could represent the location of the roots of $z^5+z^2-z+c=0$ in the complex plane, where $c \in \mathbb{R}$?

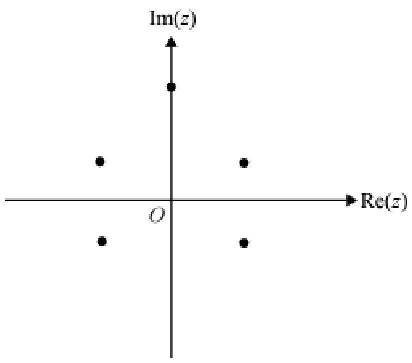
(A)



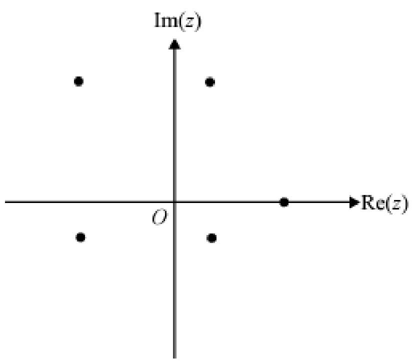
(B)



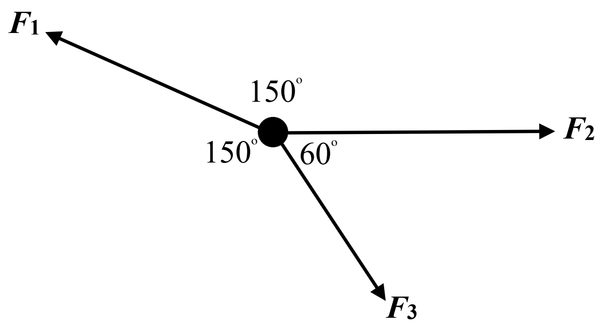
(C)



(D)



6 Three forces F_1, F_2 and F_3 act on a particle as shown in the diagram below



If the particle is in equilibrium, which of the following statements about the forces is true?

(A) $F_1 = F_3 = \dfrac{F_2}{\sqrt{3}}$

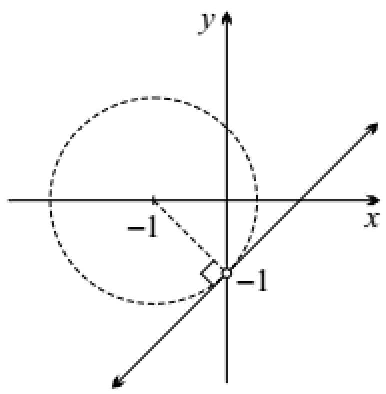
(B) $F_2 = F_3 = \dfrac{F_1}{\sqrt{3}}$

(C) $F_2 = F_3 = \sqrt{3}\,F_1$

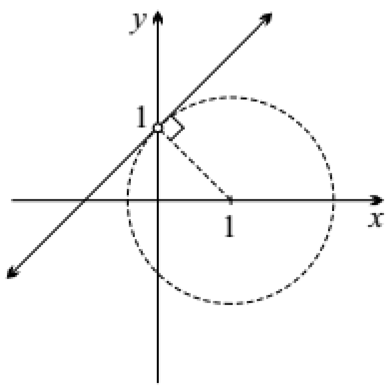
(D) $F_1 = F_2 = \sqrt{3}\,F_3$

7 If $\omega = \dfrac{z+1}{z+i}$ and ω is purely imaginary, what is the locus of z ?

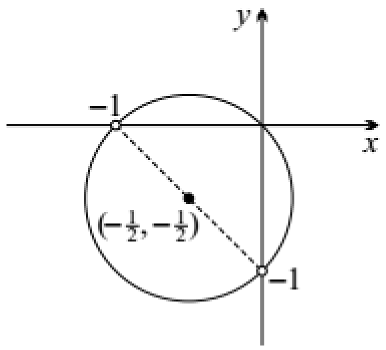
(A)



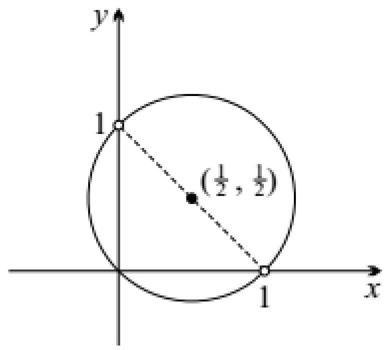
(B)



(C)



(D)



8 If the points A, B and C are such that $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$, which of the following statements **MUST** be true?

(A) Either $\overrightarrow{AB}$ or $\overrightarrow{BC}$ is the zero vector.

(B) $|\overrightarrow{AB}| = |\overrightarrow{BC}|$

(C) A, B and C are collinear.

(D) $\text{proj}_{\overrightarrow{BC}}\overrightarrow{AC} = \overrightarrow{BC}$

9 A sufficient condition for a ΔABC to be right-angled is that $a^2 + b^2 = c^2$.

Which of the following is an equivalent statement?

(A) If ΔABC is right-angled, then $a^2 + b^2 = c^2$.

(B) If $a^2 + b^2 = c^2$, then ΔABC is right-angled.

(C) If $a^2 + b^2 \neq c^2$, then ΔABC is not right-angled.

(D) ΔABC is right-angled if and only if $a^2 + b^2 = c^2$.

10 The displacement of a particle moving along the x-axis is given by

$$x = 2\cos(nt) - \sin[(2n-1)t] \text{ , where } n \neq 1$$

What is the value of n, if the motion of the particle is not simple harmonic motion?

(A) $n = \frac{1}{2}$

(B) $n = \frac{1}{3}$

(C) $n = \frac{1}{4}$

(D) $n = 0$

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your NESA#. Extra paper is available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (14 marks) Use the pages labelled Question 11 in the answer booklet

(a) Consider the complex numbers $z = 2 + i$ and $w = 3 - 2i$. Find, in Cartesian form, the values of

(i) $\dfrac{1}{w}$ *1*

(ii) $z + \overline{w}$ *1*

(iii) zw *1*

(iv) $|z - w|$ *1*

(b) Find $\displaystyle\int \frac{dx}{\sqrt{2 + 2x - x^2}}$. *2*

(c) (i) Write the complex number $\sqrt{2} - i\sqrt{2}$ in exponential form. *2*

(ii) Hence find the exact value of $\left(\sqrt{2} - i\sqrt{2}\right)^9$, giving your answer in the form $a + ib$. *2*

Question 11 continues on page 7

Question 11 (continued)

(d) A particle is moving along a straight line and is released from rest at a point 2 metres to the right of the origin. It is known that the particle moves in simple harmonic motion described by the equation

$$\ddot{x} = -4(x-5)$$

(i) Find a possible displacement-time equation that would describe the particle's motion. *2*

(ii) Determine the particle's greatest speed. *1*

(iii) How long does it take for the particle to complete one oscillation? *1*

End of Question 11

Question 12 (15 marks) Use the pages labelled Question 12 in the answer booklet

(a) Consider the two lines in three dimensions given by

$$\underset{\sim}{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} \quad \text{and} \quad \underset{\sim}{r} = \begin{pmatrix} 4 \\ -2 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

(i) By equating components, show that these two lines never intersect. *2*

(ii) Explain why these lines are not parallel. *1*

(b) Solve the quadratic equation $z^2 - 3z + (3+i) = 0$. *3*

(c) Find $\displaystyle\int \frac{dx}{(x+2)\sqrt{x^2+4x-5}}$. *3*

(d) (i) Express the roots of $z^5 - 1 = 0$ in polar form. *2*

(ii) Find real numbers a and b such that *2*

$$x^4 + x^3 + x^2 + x + 1 = (x^2 + ax + 1)(x^2 + bx + 1)$$

(iii) Hence find the exact value of $\cos\dfrac{2\pi}{5}$. *2*

Question 13 (15 marks) Use the pages labelled Question 13 in the answer booklet

(a) A particle is moving along the x-axis. Initially the particle is at the origin and its velocity is given by

$$v = (k - x)^2$$

for some positive constant k, and where x is its displacement from the origin, measured in metres after t seconds.

(i) Show that $x < k$ for all values of t. *3*

(ii) Deduce that the particle is always moving to the right and slowing down. *2*

(b) (i) Show that $\displaystyle\int_0^1 \frac{5 - 5x^2}{(1 + 2x)(1 + x^2)}\,dx = \frac{1}{2}\left(\pi + \ln\frac{27}{16}\right)$. *3*

(ii) Hence find $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + 2\sin x + \cos x}\,dx$. *3*

(c) The distinct points P, Q, R and S in the Argand diagram, lie on a circle of radius a units, centred at the origin, and are represented by the complex numbers p, q, r and s respectively.

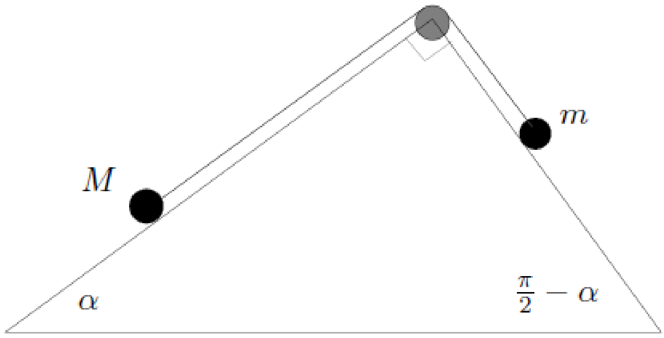
(i) Show that $pq = \dfrac{a^2(p - q)}{\overline{q} - \overline{p}}$. *2*

(ii) Deduce that if the chords PQ and RS are perpendicular, then $pq + rs = 0$. *2*

Question 14 (14 marks) Use the pages labelled Question 14 in the answer booklet

(a) A triangular wedge is fixed to a horizontal surface. The base angles of the wedge are α and $\frac{\pi}{2}-\alpha$. *4*

Two particles of mass M and m, lie on different faces of the wedge, and are connected by a light string which passes over a small pulley at the apex of the wedge, as shown in the diagram.



The contacts between the particles and the wedge are smooth (i.e. you may assume that friction is negligible).

Show that if $\tan\alpha > \frac{m}{M}$, the particle of mass M will accelerate down the face of the wedge.

(b) (i) For all real numbers $a,\ b \geq 0$, prove that $a+b \geq 2\sqrt{ab}$. *1*

(ii) Solve $(2^{2x}+1)(2^{2y}+2)(2^{2z}+8)=2^{5+x+y+z}$. *3*

(c) The distinct points $O(0,0,0)$, $A(a^3,a^2,a)$ and $B(b^3,b^2,b)$ with $a > b > 0$ lie in three-dimensional space.

(i) Prove that A and B cannot both lie on a sphere centred at O. *3*

(ii) Given that a and b can vary with $ab = 1$, show that $0 < \angle AOB < \frac{\pi}{2}$. *3*

Question 15 (15 marks) Use the pages labelled Question 15 in the answer booklet

(a) Using the substitution $u = -x$, or otherwise, evaluate $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}\,dx$. *3*

(b) A particle of mass m kg is dropped from rest in a medium where the resistance is mkv^2 newtons where the speed of the particle is v m/s and the terminal velocity is W m/s.

After t seconds, the particle has fallen x metres, and the acceleration due to gravity is g m/s^2.

(i) With the use of a force diagram, explain why $\ddot{x} = \dfrac{g}{W^2}(W^2 - v^2)$. *2*

(ii) Show that *4*

$$Wt - x = \frac{W^2}{g}\ln\left(1 + \frac{v}{W}\right)$$

(*Note: you may use* $\displaystyle\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$ *without proof)*

(c)(i) Prove that the cube root of any irrational number, is also an irrational number. *3*

(ii) Let $u_n = 5^{\frac{1}{3^n}}$. Given that $\sqrt[3]{5}$ is an irrational number, prove by induction that u_n is an irrational number, for all positive integer values of n. *3*

Question 16 (17 marks) Use the pages labelled Question 16 in the answer booklet

(a) Let $I_n = \int_0^{2\pi} e^x \cos nx \; dx$ where $n \in \mathbb{Z}^+$

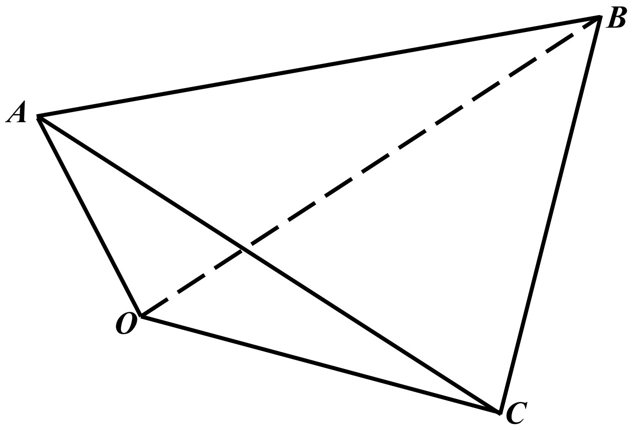
(i) Show that $I_n = \dfrac{1}{n^2+1}(e^{2\pi} - 1)$. *3*

(ii) Find the exact value of $\int_0^{2\pi} e^x \cos x \cos 6x \; dx$. *3*

Question 16 continues on page 13

Question 16 (continued)

(b) A tetrahedron is called isosceles if each pair of edges, which do not share a vertex, are equal. i.e. $AB = OC$, $BC = OA$ and $AC = OB$



Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OC} = \underset{\sim}{c}$

(i) Explain why all four faces of an isosceles tetrahedron are congruent. *1*

(ii) Show that $2\underset{\sim}{b}\cdot\underset{\sim}{c} = |\underset{\sim}{b}|^2 + |\underset{\sim}{c}|^2 - |\underset{\sim}{a}|^2$ *1*

(iii) Show that $\underset{\sim}{a}\cdot(\underset{\sim}{b} + \underset{\sim}{c}) = |\underset{\sim}{a}|^2$ *1*

(iv) By considering the length of the vector $\underset{\sim}{a} - \underset{\sim}{b} - \underset{\sim}{c}$, or otherwise, show that in an isosceles tetrahedron, none of the angles between pairs of edges which share a vertex, can be obtuse. *3*

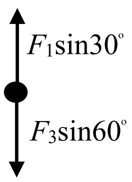
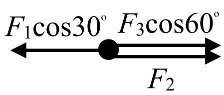
(v) Explain why it is not possible for any of the angles between pairs of edges in an isosceles tetrahedron to be a right angle. *2*

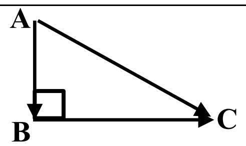
(c) Prove *3*

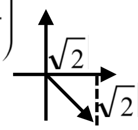
$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \ldots + \frac{1}{\sqrt{2023}+\sqrt{2024}} > 22$$

End of paper

BAULKHAM HILLS HIGH SCHOOL
2024 YEAR 12 EXTENSION 2 TRIAL HSC SOLUTIONS

Solution	Marks	Comments
SECTION I		
1. B – P: the volleyball team doesn't play well Q: they are not training enough $(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$ $\therefore$ C is the contrapositive of the statement	**1**	
2. A – $\int f(x)\sin x\,dx$ $\quad u = f(x) \quad v = -\cos x$ $du = f'(x)dx \quad dv = \sin x\,dx$ $= -f(x)\cos x + \int f'(x)\cos x\,dx$ $\therefore f'(x) = 3x^2$ $f(x) = x^3$	**1**	
3. D – A: $x + iy = i(x - iy)$ $= ix + y$ $(x-y)+(x-y)i = 0$ $\therefore \quad y = x$ which passes through (0,0) B: $2x = 0$ $x = 0$ which passes through (0,0) C: $x - 2y = 0$ $y = \frac{x}{2}$ which passes through (0,0) D: $x + y = 1$ $y = 1 - x$ which does $\boldsymbol{NOT}$ pass through (0,0)	**1**	
4. A – $\frac{e^{-\frac{i\pi}{2}}}{e^{\frac{i\pi}{6}}} = e^{-\frac{2i}{3}}$ $= \cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)$ $= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$	**1**	
5. B - As the coefficients are all real, complex roots must appear in conjugate pairs, this eliminates options A and D. As the polynomial is of an odd order, then there must be at least one real root, which eliminates option C. Thus, option B is the only possible option	**1**	
6. B – Resolving forces vertically  $F_1\sin 30° = F_3\sin 60°$ $F_1 = \sqrt{3}F_3$ $\frac{F_1}{\sqrt{3}} = F_3$ Resolving forces horizontally  $F_1\cos 30° = F_2 + F_3\cos 60°$ $\sqrt{3}F_1 = 2F_2 + F_3$ $3F_2 = 2F_2 + F_3$ $F_2 = F_3$	**1**	
7. C – $\text{Re}(\omega) = 0 \Rightarrow \arg\left(\frac{z+1}{z-i}\right) = \pm\frac{\pi}{2}$ $\therefore$ the locus of z is a circle with diameter joining $(-1,0)$ and $(0,-1)$ but not including these points	**1**	

Solution	Marks	Comments
8. D – $\overrightarrow{AB}\cdot\overrightarrow{BC}=0\Rightarrow\overrightarrow{AB}\perp\overrightarrow{BC}$ $\therefore$ $\overrightarrow{BC}$ is the projection of $\overrightarrow{AC}$ on $\overrightarrow{BC}$ 	**1**	
9. B – In $P\Rightarrow Q$, P is a sufficient condition for Q Q is a necessary condition for P $\therefore$ P: $a^2+b^2=c^2$ Q: ΔABC is right angled thus B is the correct option NOTE: (A) $\Leftrightarrow$ (C), as they are contrapositives (D) is an equivalence statement i.e. $P\Leftrightarrow Q$ and thus both P and Q would be necessary conditions	**1**	
10. C – $x=2\cos(nt)-\sin[(2n-1)t]$ $\dot{x}=-2n\sin(nt)-(2n-1)\cos[(2n-1)t]$ $\ddot{x}=-2n^2\cos(nt)+(2n-1)^2\sin[(2n-1)t]$ For SHM, $\ddot{x}=-n^2x$, and as $-2n^2$ and $(2n-1)^2$ do not have a common factor, either $2n^2=0$ or $(2n-1)^2=0$ $n=0$ $\quad$ $n=\frac{1}{2}$ Additionally, $-n^2x=-2n^2\cos(nt)+n^2\sin[(2n-1)t]$, so it will be SHM if $n^2=(2n-1)^2$ $n^2=4n^2-4n+1$ $3n^2-4n+1=0$ $(3n-1)(n-1)=0$ $n=\frac{1}{3}$ or $n=1$ not a solution Thus $n=\frac{1}{4}$ is the value that does not produce an equation of motion that is SHM	**1**	
SECTION II		
QUESTION 11		
11(a) (i) $\frac{1}{w}=\frac{\overline{w}}{\lvert w\rvert^2}$ $=\frac{3+2i}{3^2+(-2)^2}$ $=\frac{3}{13}+\frac{2}{13}i$	**1**	**1 mark** • Correct answer
11 (a) (ii) $z+\overline{w}=2+i+3+2i$ $=5+3i$	**1**	**1 mark** • Correct answer
11 (a) (iii) $zw=(2+i)(3-2i)$ $=6-4i+3i+2$ $=8-i$	**1**	**1 mark** • Correct answer
11 (a) (ii) $\lvert z-w\rvert=\lvert -1-3i\rvert$ $=\sqrt{(-1)^2+(-3)^2}$ $=\sqrt{10}$	**1**	**1 mark** • Correct answer
11 (b) $\int\frac{dx}{\sqrt{2+2x-x^2}}$ $=\int\frac{dx}{\sqrt{3-(x-1)^2}}$ $=\sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right)+c$	**2**	**2 marks** • Correct solution **1 mark** • Completes the square in the denominator

Solution	Marks	Comments
11 (c) (i) $\lvert\sqrt{2}-i\sqrt{2}\rvert=\sqrt{\left(\sqrt{2}\right)^2+\left(-\sqrt{2}\right)^2}$ $=\sqrt{4}$ $=2$ $\text{Arg}\left(\sqrt{2}-i\sqrt{2}\right)=\tan^{-1}\left(\frac{-\sqrt{2}}{\sqrt{2}}\right)$ $=-\frac{\pi}{4}$  $\therefore\sqrt{2}-i\sqrt{2}=2e^{-\frac{i\pi}{4}}$	2	**2 marks** • Correct solution **1 mark** • Obtains correct modulus or argument
11 (c) (ii) $\left(\sqrt{2}-i\sqrt{2}\right)^9=\left(2e^{-\frac{i\pi}{4}}\right)^9$ $=2^9e^{-\frac{9i\pi}{4}}$ $=512\left[\cos\left(-\frac{9\pi}{4}\right)+i\sin\left(-\frac{9\pi}{4}\right)\right]$ $=512\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)$ $=256\sqrt{2}-256\sqrt{2}\,i$	2	**2 marks** • Correct solution **1 mark** • Attempts to apply de Moivre's theorem or equivalent merit
11 (d) (i) $\ddot{x}=-n^2(x-c)\Rightarrow x=a\cos nt+c$ $c=5$ $\quad n^2=4$ $n=2\ \ (n>0)$ $x=a\cos 2t+5$ when $t=0$, $x=2$ $2=a+5$ $a=-3$ $\therefore x=5-3\cos 2t$	2	**2 marks** • Correct solution **1 mark** • Finds two of *a, n* and *c*
11 (d) (ii) $x=5-3\cos 2t$ $\dot{x}=6\sin 2t$ $\therefore$ greatest speed is 6 m/s	1	**1 mark** • Correct answer
11 (d) (iii) $T=\frac{2\pi}{2}$ $=\pi$ $\therefore$ it takes the particle π seconds to complete one oscillation	1	**1 mark** • Correct answer
QUESTION 12		
12 (a) (i) $\begin{pmatrix}1\\0\\2\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\-3\end{pmatrix}=\begin{pmatrix}4\\-2\\9\end{pmatrix}+\mu\begin{pmatrix}1\\2\\-2\end{pmatrix}\Rightarrow$ $1+2\lambda=4+\mu\ \ \ldots (1)$ $2\lambda=-2+2\mu\ldots (2)$ $2-3\lambda=9-2\mu\ \ \ldots (3)$ substituting (2) into (1) $1-2+2\mu=4+\mu$ $\mu=5\ \ \therefore\lambda=4$ substituting $\mu=5$ into (3) $2-3\lambda=9-10$ $\lambda=1\neq 4$ $\therefore$ the two lines do not intersect	2	**2 marks** • Correct solution **1 mark** • Obtains a set of values for λ and μ
12 (a) (ii) $\begin{pmatrix}2\\2\\-3\end{pmatrix}\neq k\begin{pmatrix}1\\2\\-2\end{pmatrix}$ where $k\in\mathbb{Z}$ i.e. the direction vectors are not scalar multiples of each other. $\therefore$ the lines are not parallel	1	**1 mark** • Correct explanation
12(b) $z=\frac{3\pm\sqrt{9-4(3+i)}}{2}$ $=\frac{3\pm\sqrt{-3-4i}}{2}$ $=\frac{3\pm(1-2i)}{2}$ $z=2-i$ or $1+i$ $\sqrt{-3-4i}=a+ib$ $a^2-b^2=-3$ $a^2+b^2=5$ $2a^2=2$ $a=\pm1\ \ ,\ \ b=\mp2$ $\sqrt{-3-4i}=\pm(1-2i)$	3	**3 marks** • Correct solution **2 marks** • Finds $\sqrt{-3-4i}$ or equivalent merit **1 mark** • Completes the square or uses the quadratic formula

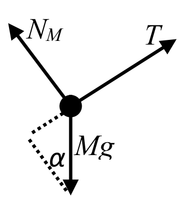
Solution	Marks	Comments
12(c) $\displaystyle\int \frac{dx}{(x+2)\sqrt{x^2+4x-5}}$ $x+2=3\sec\theta \Rightarrow \sec\theta=\frac{x+2}{3}$ $dx=3\sec\theta\tan\theta d\theta$ $=\displaystyle\int \frac{3\sec\theta\tan\theta d\theta}{3\sec\theta\sqrt{9\sec^2\theta-9}}$ $=\displaystyle\int \frac{\tan\theta d\theta}{3\tan\theta}$ $=\displaystyle\frac{1}{3}\int d\theta$ $=\frac{1}{3}\theta+c$ $=\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{x^2+4x-5}}{3}\right)+c$ *Note: whilst* $\frac{1}{3}\sec^{-1}\left(\frac{x+2}{3}\right)$ *is correct for* $x\ge 1$, *the correct answer for* $x\le -5$ *would be* $-\frac{1}{3}\sec^{-1}\left(\frac{x+2}{3}\right)$	3	**3 marks** • Correct solution **2 marks** • Transforms the integrand via a suitable substitution or equivalent merit **1 mark** • Completes the square in the denominator *Note: no penalty for the answer* $\frac{1}{3}\sec^{-1}\left(\frac{x+2}{3}\right)$
12 (d) (i) $z^5-1=0$ $z^5=1$ $z=\text{cis}\left(\frac{2\pi k}{5}\right)$ where $k\in\mathbb{Z}$ $z=\cos 0+i\sin 0$, $\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}$, $\cos\left(-\frac{2\pi}{5}\right)+i\sin\left(-\frac{2\pi}{5}\right)$, $\cos\frac{4\pi}{5}+i\sin\frac{4\pi}{5}$, $\cos\left(-\frac{4\pi}{5}\right)+i\sin\left(-\frac{4\pi}{5}\right)$	2	**2 marks** • Correct solution **1 mark** • Uses de Moivre's theorem to generate the fifth roots of unity or equivalent merit *Note: no penalty is cis0 is written as 1*
12 (d) (ii) $x^4+x^3+x^2+x+1=(x^2+ax+1)(x^2+bx+1)$ $=x^4+(a+b)x^3+(2+ab)x^2+(a+b)x+1$ by equating coefficients $a+b=1$ $2+ab=1$ $ab=-1$ $a-\frac{1}{a}=1$ $b=-\frac{1}{a}$ $a^2-a-1=0$ $a=\frac{1\pm\sqrt{5}}{2}$ thus $a=\frac{1-\sqrt{5}}{2}$ and by symmetry $b=\frac{1+\sqrt{5}}{2}$	2	**2 marks** • Correct solution **1 mark** • Finds two relationships between a and b or equivalent merit
12 (d) (iii) $\frac{x^5-1}{x-1}=x^4+x^3+x^2+x+1$ So the roots of $x^4+x^3+x^2+x+1=0$ are the same as the roots of $x^5-1=0$, excluding $x=1$. Additionally, as the coefficients of x^2+ax+1 are real, then all complex roots must appear in conjugate pairs. Since $\frac{2\pi}{5}$ is acute, $\cos\frac{2\pi}{5}>0$ $\therefore$ $-a=2\cos\frac{2\pi}{5}$ (sum of the roots) $2\cos\frac{2\pi}{5}=\frac{-1+\sqrt{5}}{2}$ $\cos\frac{2\pi}{5}=\frac{\sqrt{5}-1}{4}$	2	**2 marks** • Correct solution **1 mark** • Connects the roots of $x^5-1=0$ with the roots of $x^4+x^3+x^2+x+1=0$ or equivalent merit

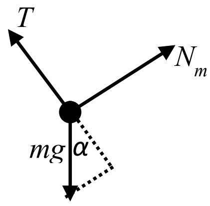
Solution	Marks	Comments
QUESTION 13		
13 (a) (i) $\dfrac{dx}{dt}=(k-x)^2$ $\displaystyle\int_0^x \frac{dx}{(k-x)^2}=\int_0^t dt$ $t=\left[\dfrac{1}{k-x}\right]_0^x$ $=\dfrac{1}{k-x}-\dfrac{1}{k}$ $t+\dfrac{1}{k}=\dfrac{1}{k-x}$ $\dfrac{kt+1}{k}=\dfrac{1}{k-x}$ $k-x=\dfrac{k}{kt+1}$ $x=k-\dfrac{k}{kt+1}$ $\therefore x<k \qquad \left(k>0 \wedge t>0 \Rightarrow \dfrac{k}{kt+1}>0\right)$	3	**3 marks** • Correct solution **2 marks** • Finds x as a function of t or equivalent merit **1 mark** • Finds an expression for t as a function of x or equivalent merit
13 (a) (ii) $v=(k-x)^2>0 \qquad (x\neq k)$ Thus the particle is always moving to the right $\ddot{x}=v\dfrac{dv}{dx}$ $=(k-x)^2\times 2(k-x)(-1)$ $=-2(k-x)^3$ since $x<k$, $\ddot{x}<0$ as $\ddot{x}$ and v are in opposite directions, the particle is slowing down.	2	**2 marks** • Correct solution **1 mark** • Notes that $v>0$ • finds $\ddot{x}$ in terms of x or equivalent merit
13 (b) (i) $\dfrac{5-5x^2}{(1+2x)(1+x^2)}\equiv\dfrac{A}{1+2x}+\dfrac{Bx+C}{1+x^2}$ $x=-\dfrac{1}{2}$ $A=\dfrac{5-5\left(-\frac{1}{2}\right)^2}{\left(1+\left(-\frac{1}{2}\right)^2\right)}$ $=\dfrac{5-\frac{5}{4}}{1+\frac{1}{4}}$ $=3$ $x=i$ $Bi+C=\dfrac{5-5i^2}{1+2i}$ $=\dfrac{10}{1+2i}\times\dfrac{1-2i}{1-2i}$ $=\dfrac{10-20i}{5}$ $=2-4i$ $\therefore \quad B=-4,\ C=2$ $\displaystyle\int_0^1 \frac{5-5x^2}{(1+2x)(1+x^2)}dx=\int_0^1\left[\frac{3}{1+2x}-\frac{4x}{1+x^2}+\frac{2}{1+x^2}\right]dx$ $=\left[\dfrac{3}{2}\ln\lvert 1+2x\rvert-2\ln\lvert 1+x^2\rvert+2\tan^{-1}x\right]_0^1$ $=\dfrac{3}{2}\ln 3-2\ln 2+2\left(\dfrac{\pi}{4}\right)-0$ $=\dfrac{1}{2}\ln\dfrac{3^3}{2^4}+\dfrac{\pi}{2}$ $=\dfrac{1}{2}\left(\ln\dfrac{27}{16}+\pi\right)$	3	**3 marks** • Correct solution **2 marks** • Finds the primitive **1 mark** • Decomposes the integrand into partial fractions

Solution	Marks	Comments
13 (b) (ii) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+2\sin x+\cos x}dx$ $t=\tan\frac{x}{2}$ $dx=\frac{2dt}{1+t^2}$ when $x=0$, $t=0$ when $x=\frac{\pi}{2}$, $t=1$ $\displaystyle=\int_0^1\left(\frac{\frac{1-t^2}{1+t^2}}{1+\frac{4t}{1+t^2}+\frac{1-t^2}{1+t^2}}\times\frac{2dt}{1+t^2}\right)$ $\displaystyle=\int_0^1\frac{2-2t^2}{(1+t^2+4t+1-t^2)(1+t^2)}dt$ $\displaystyle=\int_0^1\frac{2-2t^2}{(2+4t)(1+t^2)}dt$ $\displaystyle=\frac{1}{5}\int_0^1\frac{5-5t^2}{(1+2t)(1+t^2)}dt$ $\displaystyle=\frac{1}{10}\left(\ln\frac{27}{16}+\pi\right)$	**3**	**3 marks** • Correct solution **2 marks** • Transforms the integrand into a multiple of part (i) **1 mark** • Substitutes t-results into the integrand
13 (c) (i) circle has equation $\lvert z\rvert=a$ $z\overline{z}=a^2$ $\therefore p\overline{p}=q\overline{q}=a^2$ $a^2(p-q)=a^2p-a^2q$ $=q\overline{q}p-p\overline{p}q$ $=pq\left(\overline{q}-\overline{p}\right)$ $pq=\dfrac{a^2(p-q)}{\overline{q}-\overline{p}}$	**2**	**2 marks** • Correct solution **1 mark** • Establishes $p\overline{p}=q\overline{q}=a^2$ or equivalent merit
13 (c) (ii) if $PQ\perp RS$ then $q-p=ki(r-s)$ and $\overline{q-p}=\overline{ki(r-s)}$ $\underset{\sim}{\overline{q}}-\overline{p}=-\underset{\sim}{ki}\left(\overline{r}-\overline{s}\right)$ $pq+rs=\dfrac{a^2(p-q)}{\overline{q}-\overline{p}}+\dfrac{a^2(s-r)}{\overline{r}-\overline{s}}$ $=\dfrac{a^2i(s-r)}{-i\left(\overline{r}-\overline{s}\right)}+\dfrac{a^2(s-r)}{\left(\overline{r}-\overline{s}\right)}$ $=-\dfrac{a^2(s-r)}{\left(\overline{r}-\overline{s}\right)}+\dfrac{a^2(s-r)}{\left(\overline{r}-\overline{s}\right)}$ $=0$	**2**	**2 marks** • Correct solution **1 mark** • Establishes $q-p=(r-\mathrm{s})$ or equivalent merit

<table>
<tr><th>Solution</th><th>Marks</th><th>Comments</th></tr>
<tr><th colspan="3">QUESTION 14</th></tr>
<tr><td>

14 (a) forces on M forces on m





Resolving forces down the plane

$M\ddot{x} = Mg\sin\alpha - T$

Resolving forces up the plane

$m\ddot{y} = T - mg\cos\alpha$

as both particles are connected by a string, they have the same acceleration

$$Ma = Mg\sin\alpha - T$$
$$ma = T - mg\cos\alpha$$
$$(M+m)a = g(M\sin\alpha - m\cos\alpha)$$
$$a = \frac{g(M\sin\alpha - m\cos\alpha)}{(M+m)}$$

mass M will slide down the plane when $a > 0$

i.e. $M\sin\alpha - m\cos\alpha > 0$

$$M\sin\alpha > m\cos\alpha$$
$$\frac{\sin\alpha}{\cos\alpha} > \frac{m}{M}$$
$$\tan\alpha > \frac{m}{M}$$

</td><td>4</td><td>

4 marks
- Correct solution

3 marks
- Finds an expression for a involving M, m and α

2 marks
- Links the equations of motion for the two particles by eliminating T or equivalent merit

1 mark
- Finds an equation of motion for either particle

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14 (b) (i) $\left(\sqrt{a} - \sqrt{b}\right)^2 \geq 0$

$$a - 2\sqrt{ab} + b \geq 0$$
$$a + b \geq 2\sqrt{ab}$$

</td><td>1</td><td>

1 mark
- Correct solution

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14 (b) (ii)

$2^{2x} + 1 \geq 2\sqrt{2^{2x}}$
$= 2 \times 2^x$
$= 2^{x+1}$

$2^{2y} + 2 \geq 2\sqrt{2^{2y+1}}$
$= 2 \times 2^{y+\frac{1}{2}}$
$= 2^{y+\frac{3}{2}}$

$2^{2z} + 8 \geq 2\sqrt{2^{2z+3}}$
$= 2 \times 2^{z+\frac{3}{2}}$
$= 2^{z+\frac{5}{2}}$

$$\therefore (2^{2x}+1)(2^{2y}+2)(2^{2z}+8) \geq 2^{x+1} \times 2^{y+\frac{3}{2}} \times 2^{z+\frac{5}{2}}$$
$$= 2^{5+x+y+z}$$

$a + b \geq 2\sqrt{ab}$ and equality occurs when $a = b$

$2^{2x} = 1$, $2x = 0$, $x = 0$

$2^{2y} = 2$, $2y = 1$, $y = \frac{1}{2}$

$2^{2z} = 8$, $2z = 3$, $z = \frac{3}{2}$

$$\therefore x = 0,\ y = \frac{1}{2},\ z = \frac{3}{2}$$

</td><td>3</td><td>

3 marks
- Correct solution

2 marks
- Establishes the required inequality or equivalent merit

1 mark
- Attempts to use part (i) in the solution

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14 (c) (i) If A and B do lie on a sphere then

$$\left|\begin{pmatrix} a^3 \\ a^2 \\ a \end{pmatrix}\right| = \left|\begin{pmatrix} b^3 \\ b^2 \\ b \end{pmatrix}\right|$$
$$a^6 + a^4 + a^2 = b^6 + b^4 + b^2$$
$$a^6 - b^6 + a^4 - b^4 + a^2 - b^2 = 0$$
$$(a^2 - b^2)(a^4 + a^2b^2 + b^4) + (a^2 - b^2)(a^2 + b^2) + (a^2 - b^2) = 0$$
$$(a^2 - b^2)(a^4 + a^2b^2 + b^4 + a^2 + b^2 + 1) = 0$$

$a^2 - b^2 = 0$

$a = b \quad (a > 0,\ b > 0)$

No solution as A and B are distinct points

$a^4 + a^2b^2 + b^4 + a^2 + b^2 + b^2 + 1 = 0$

No solution as $a > 0$ and $b > 0$

$\therefore$ A and B cannot lie on a sphere centred O

</td><td>3</td><td>

3 marks
- Correct solution

2 marks
- Equates the two magnitudes and finds an algebraic expression with $(a-b)^2$ as a common factor

1 mark
- Finds the magnitude of either vector or equivalent merit

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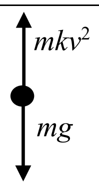
Solution	Marks	Comments
14 (c) (ii) $\cos\ \angle AOB = \dfrac{a^3b^3 + a^2b^2 + ab}{\sqrt{a^6+a^4+a^2}\sqrt{b^6+b^4+b^2}}$ $\le \dfrac{3}{\sqrt{3}\ \sqrt[3]{a^{12}}\ \sqrt{3}\ \sqrt[3]{b^{12}}}$ $\quad (AM \ge GM)$ $= \dfrac{3}{3a^4b^4}$ $=1$ $0 < \cos\ \angle AOB < 1$ (as equality ocurs when $a=b$, however A and B are distinct points) $\therefore 0 < \angle AOB < \dfrac{\pi}{2}$	**3**	**3 marks** • Correct solution **2 marks** establishes $\angle AOB \le 1$ **1 mark** • Uses the dot product to find an expression for $\angle AOB$
QUESTION 15		
15 (a) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}dx = -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2(-u)}{1+2^{-u}}du$ $\quad u=-x$ when $x=-\dfrac{\pi}{2}$ $u=\dfrac{\pi}{2}$ $du=-dx$ when $x=\dfrac{\pi}{2}$ $u=-\dfrac{\pi}{2}$ $\displaystyle = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 u}{1+2^{-u}} \times \frac{2^u}{2^u}du$ $\displaystyle = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2^u \sin^2 u}{2^u+1}du$ $\displaystyle \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(1+2^x)\sin^2 x}{1+2^x}dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\ dx$ $\displaystyle \therefore 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}dx = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1-\cos 2x)dx$ $\displaystyle \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}dx = \frac{1}{4}\left[x-\frac{1}{2}\sin 2x\right]_{\frac{\pi}{2}}^{-\frac{\pi}{2}}$ $= \dfrac{\pi}{8} - 0 + \dfrac{\pi}{8} + 0$ $= \dfrac{\pi}{4}$	**3**	**3 marks** • Correct solution **2 marks** • Simplifies the integrand into a simple trig integral or equivalent merit **1 mark** • Uses the given substitution to transform the integrand
15 (b) (i)  $m\ddot{x} = mg - mkv^2$ $\dot{x} = g - kv^2$ terminal velocity occurs when $\ddot{x}=0$ $g - kW^2 = 0$ $k = \dfrac{g}{W^2}$ $\therefore \quad \ddot{x} = g - \dfrac{gv^2}{W^2}$ $= \dfrac{g}{W^2}(W^2 - v^2)$	**2**	**2 marks** • Correct solution **1 mark** • Shows the two forces on the particle in a force diagram • Finds k

Solution	Marks	Comments
15(b) (ii) $v\frac{dv}{dx}=\frac{g}{W^2}(W^2-v^2)$ $\quad$ $\frac{dv}{dt}=\frac{g}{W^2}(W^2-v^2)$ $\frac{W^2}{g}\int_0^v \frac{v\,dv}{W^2-v^2}=\int_0^x dx$ $\quad$ $\frac{W^2}{g}\int_0^v \frac{dv}{W^2-v^2}=\int_0^t dt$ $x=-\frac{W^2}{2g}\left[\ln(W^2-v^2)\right]_0^v$ $\quad$ $t=\frac{W^2}{g}\left[\frac{1}{2W}\ln\left(\frac{W+v}{W-v}\right)\right]_0^v$ $=-\frac{W^2}{2g}\ln\left(\frac{W^2-v^2}{W^2}\right)$ $\quad$ $=\frac{W}{2g}\ln\left(\frac{W+v}{W-v}\right)$ $Wt-x=\frac{W^2}{2g}\ln\left(\frac{W+v}{W-v}\right)+\frac{W^2}{2g}\ln\frac{(W^2-v^2)}{W^2})$ $=\frac{W^2}{2g}\ln\left(\frac{W+v}{W-v}\times\frac{(W-v)(W+v)}{W^2}\right)$ $=\frac{W^2}{2g}\ln\frac{(W+v)^2}{W^2}$ $=\frac{W^2}{g}\ln\left(1+\frac{v}{W}\right)$	**4**	**4 marks** • Correct solution **3 marks** • Attempts to link the two acceleration equations and makes significant progress towards the final solution **2 marks** • Finds a primitive for x and t in terms of v **1 mark** • Finds a primitive for x or t in terms of v
15(c) (i) Let r be an irrational number and that $\sqrt[3]{r}=\frac{p}{q}$ where $(p \wedge q \in \mathbb{Z}^+) \wedge q \neq 0$ i.e. $\sqrt[3]{r}$ is rational $\quad$ also $p \wedge q$ are coprime $r=\frac{p^3}{q^3}$ $rq^3=p^3$ $\therefore r$ is a factor of p^3 however rq^3 is irrational as r is irrational, yet p^3 is rational ($p \in \mathbb{Z}$) $\therefore \sqrt[3]{r}$ is irrational, by contradiction	**3**	**3 marks** • Correct solution **2 marks** • Shows that r is a factor of p^3 or equivalent merit **1 mark** • Attempts proof by contradiction or equivalent merit
15(c) (ii) $u_n=5^{\frac{1}{3^n}}$, prove that u_n is an irrational number for $n \in \mathbb{Z}^+$ When $n = 1$ $u_1=5^{\frac{1}{3}}$ $=\sqrt[3]{5}$ Which is irrational Hence the result is true for $n = 1$ Assume the result is true for $n = k$ where $k \in \mathbb{Z}^+$ i.e. $u_k=5^{\frac{1}{3^k}}$ is irrational Prove the result is true for $n = k + 1$ i.e. $u_{k+1}=5^{\frac{1}{3^{k+1}}}$ is irrational ***PROOF:*** $u_{k+1}=5^{\frac{1}{3^{k+1}}}$ $=5^{\frac{1}{3^k}\times\frac{1}{3}}$ $=u_k^{\frac{1}{3}}$ $=\sqrt[3]{u_k}$ from part (i) the cube root of an irrational number is also irrational $\therefore u_k$ is irrational Hence the result is true for $n = k + 1$, if it is true for $n = k$ Since the result is true for $n = 1$, then it is true $\forall\ n$ where $n \in \mathbb{Z}^+$ by induction.	**3**	There are 4 key parts of the induction; 1. Proving the result true for $n = 1$ 2. Clearly stating the assumption and what is to be proven 3. Using the assumption in the proof and acknowledges the condition for (i) 4. Correctly proving the required statement **3 marks** • Successfully does all of the 4 key parts **2 marks** • Successfully does 3 of the 4 key parts **1 mark** • Successfully does 2 of the 4 key parts

Solution	Marks	Comments
QUESTION 16		
16 (a) (i) $I_n=\int_0^{2\pi} e^x\cos nx dx$ $u=e^x$ $v=\frac{1}{n}\sin nx$ $du=e^{xdx}$ $du=\cos nx dx$ $=\left[\frac{e^x\sin nx}{n}\right]_0^{2\pi}-\frac{1}{n}\int_0^{2\pi}e^x\sin nx dx$ $=-\frac{1}{n}\int_0^{2\pi}e^x\sin nx dx$ $u=e^x$ $v=-\frac{1}{n^2}\cos nx$ $du=e^{xdx}$ $du=\sin nx dx$ $=\left[\frac{e^x\cos nx}{n^2}\right]_0^{2\pi}-\frac{1}{n^2}\int_0^{2\pi}e^x\cos nx dx$ $=\frac{e^{2\pi}-1}{n^2}-\frac{1}{n^2}I_n$ $\frac{n^2+1}{n^2}I_n=\frac{e^{2\pi}-1}{n^2}$ $I_n=\frac{1}{n^2+1}(e^{2\pi}-1)$	3	**3 marks** • Correct solution **2 marks** • Makes significant progress **1 mark** • Uses integration by parts to find a valid result
16 (a) (ii) $\int_0^{2\pi} e^x\cos x\cos 6x\ dx=\frac{1}{2}\int_0^{2\pi}e^x(\cos 5x+\cos 7x)dx$ $=\frac{1}{2}\left[\frac{1}{26}(e^{2\pi}-1)+\frac{1}{50}(e^{2\pi}-1)\right]$ $=\frac{19(e^{2\pi}-1)}{650}$	3	**3 marks** • Correct solution **2 marks** • Finds I_5 or I_7 or equivalent merit **1 mark** • Rewrites the product of two trig functions as the sum of two trig functions
16 (b) (i) A tetrahedron has six edges in total, two have length $\lvert\underset{\sim}{a}\rvert$, two have length $\lvert\underset{\sim}{b}\rvert$ and two have length $\lvert\underset{\sim}{c}\rvert$. As no two edges sharing a common vertex are equal, then the three edges of any face must be $\lvert\underset{\sim}{a}\rvert$, $\lvert\underset{\sim}{b}\rvert$ and $\lvert\underset{\sim}{c}\rvert$, and thus all four faces are congruent.	1	**1 mark** • Correct explanation
16 (b) (ii) $\lvert\overrightarrow{OA}\rvert=\lvert\overrightarrow{BC}\rvert$ $\lvert\underset{\sim}{a}\rvert^2=\lvert\underset{\sim}{c}-\underset{\sim}{b}\rvert^2$ $=(\underset{\sim}{c}-\underset{\sim}{b})\cdot(\underset{\sim}{c}+\underset{\sim}{b})$ $=\underset{\sim}{c}\cdot\underset{\sim}{c}-2\underset{\sim}{b}\cdot\underset{\sim}{c}+\underset{\sim}{b}\cdot\underset{\sim}{b}$ $=\lvert\underset{\sim}{c}\rvert^2-2\underset{\sim}{b}\cdot\underset{\sim}{c}+\lvert\underset{\sim}{b}\rvert^2$ $2\underset{\sim}{b}\cdot\underset{\sim}{c}=\lvert\underset{\sim}{b}\rvert^2+\lvert\underset{\sim}{c}\rvert^2-\lvert\underset{\sim}{a}\rvert^2$	1	**1 mark** • Correct solution
16 (c) (iii) $\underset{\sim}{a}\cdot(\underset{\sim}{b}+\underset{\sim}{c})=\underset{\sim}{a}\cdot\underset{\sim}{b}+\underset{\sim}{a}\cdot\underset{\sim}{c}$ $=\frac{1}{2}\left(\lvert\underset{\sim}{a}\rvert^2+\lvert\underset{\sim}{b}\rvert^2-\lvert\underset{\sim}{c}\rvert^2\right)+\frac{1}{2}\left(\lvert\underset{\sim}{a}\rvert^2+\lvert\underset{\sim}{c}\rvert^2-\lvert\underset{\sim}{b}\rvert^2\right)$ $=\frac{1}{2}\times 2\lvert\underset{\sim}{a}\rvert^2$ $=\lvert\underset{\sim}{a}\rvert^2$	1	**1 mark** • Correct solution

Solution	Marks	Comments
16 (b) (iv) $\lvert \underset{\sim}{a}-\underset{\sim}{b}-\underset{\sim}{c}\rvert^2 = \left\lvert \underset{\sim}{a}-(\underset{\sim}{b}+\underset{\sim}{c})\right\rvert^2$ $= \lvert\underset{\sim}{a}\rvert^2 - 2\underset{\sim}{a}\cdot(\underset{\sim}{b}+\underset{\sim}{c}) + \lvert\underset{\sim}{b}+\underset{\sim}{c}\rvert^2$ $= \lvert\underset{\sim}{a}\rvert^2 - 2\lvert\underset{\sim}{a}\rvert^2 + \lvert\underset{\sim}{b}\rvert^2 + 2\underset{\sim}{b}\cdot\underset{\sim}{c} + \lvert\underset{\sim}{c}\rvert^2$ $= -\lvert\underset{\sim}{a}\rvert^2 + \lvert\underset{\sim}{b}\rvert^2\lvert\underset{\sim}{b}\rvert^2 + \lvert\underset{\sim}{c}\rvert^2 - \lvert\underset{\sim}{a}\rvert^2 + \lvert\underset{\sim}{c}\rvert^2$ $= 2\left(\lvert\underset{\sim}{b}\rvert^2 + \lvert\underset{\sim}{c}\rvert^2 - \lvert\underset{\sim}{a}\rvert^2\right)$ $\therefore \lvert\underset{\sim}{b}\rvert^2 + \lvert\underset{\sim}{c}\rvert^2 - \lvert\underset{\sim}{a}\rvert^2 \geq 0$ now in ΔABC $\lvert\underset{\sim}{a}\rvert^2 = \lvert\underset{\sim}{b}\rvert^2 + \lvert\underset{\sim}{c}\rvert^2 - 2\lvert\underset{\sim}{b}\rvert\lvert\underset{\sim}{c}\rvert\cos\ \angle BAC$ $\cos\ \angle ABC = \dfrac{\lvert\underset{\sim}{b}\rvert^2 + \lvert\underset{\sim}{c}\rvert^2 - \lvert\underset{\sim}{a}\rvert^2}{2\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert}$ ≥ 0 $\therefore$ $\angle ABC$ cannot be obtuse, similarly neither $\angle ABC$ or $\angle BCA$ could not be obtuse either	3	**3 marks** • Correct solution **2 marks** • establishes $\angle BAC$ cannot be obtuse or equivalent merit **1 mark** • shows $\lvert\underset{\sim}{a}-\underset{\sim}{b}-\underset{\sim}{c}\rvert^2 = 2\left(\lvert\underset{\sim}{b}\rvert^2+\lvert\underset{\sim}{c}\rvert^2-\lvert\underset{\sim}{a}\rvert^2\right)$ or equivalent merit
16 (b) (v) If $\angle BAC = 90° \Rightarrow \cos\ \angle BAC = 0$ $\therefore \lvert\underset{\sim}{b}\rvert^2 + \lvert\underset{\sim}{c}\rvert^2 - \lvert\underset{\sim}{a}\rvert^2 = 0$ $\lvert\underset{\sim}{a}-\underset{\sim}{b}-\underset{\sim}{c}\rvert^2 = 0$ $\underset{\sim}{a} = \underset{\sim}{b}+\underset{\sim}{c}$ This means that $\underset{\sim}{a}$ would be the diagonal of the parallelogram $OBAC$ i.e. $OBAC$ would not be a tetrahedron Thus it is not possible for any of the angles between pairs of edges in an isosceles tetrahedron to be a right angle.	2	**2 marks** • correct explanation **1 mark** • establishes $\underset{\sim}{a} = \underset{\sim}{b}+\underset{\sim}{c}$ or equivalent merit
16 (c) $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \ldots + \frac{1}{\sqrt{2023}+\sqrt{2024}}$ $> \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \ldots + \frac{1}{\sqrt{2024}+\sqrt{2025}}$ $\therefore 2\left(\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \ldots + \frac{1}{\sqrt{2023}+\sqrt{2024}}\right)$ $> \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \ldots + \frac{1}{\sqrt{2023}+\sqrt{2024}} + \frac{1}{\sqrt{2024}+\sqrt{2025}}$ $= \frac{\sqrt{2}-1}{1} + \frac{\sqrt{3}-\sqrt{2}}{1} + \frac{\sqrt{4}-\sqrt{3}}{1} + \frac{\sqrt{6}-\sqrt{5}}{1} + \ldots + \frac{\sqrt{2024}-\sqrt{2023}}{1} + \frac{\sqrt{2025}-\sqrt{2024}}{1}$ $= \sqrt{2025} - 1$ $= 45 - 1$ $= 44$ $\therefore \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \ldots + \frac{1}{\sqrt{2023}+\sqrt{2024}} > 22$	3	**3 marks** • Correct solution **2 marks** • Creates a series of fractions such that it reduces down to two terms or equivalent merit **1 mark** • Rationalises the denominator of each fraction so that they are all 1 or – 1, or equivalent merit